3.3 Review

Sunday, February 18, 2018 5:23 PM

Name Answer Key **Date**________

Calc I H – 3.3 Review **Period** ________

For each function, find: the critical numbers *(if any)*, the open interval(s) on which the function is increasing or decreasing, and all relative extrema using the First Derivative Test.

1. $f(x)=(x-1)^2(x+2)$

$f'(x)=2(x-1)(x+2)+(x-1)^2$

$=2(x^2+x-2)+x^2-2x+1$

$=2x^2+\cancel{2x}-4+x^2-\cancel{2x}+1$

$=3x^2-3$

$f'(x)=0$

$3x^2-3=0$

$3(x^2-1)=0$

$x=\pm 1$

~~f'(x) und~~

NOT Poss.

$f(-1)=(-2)^2(1)=4$

$f(1)=0$

$f'(x)$: $+$ | $-$ | $+$

$f(x)$: $\nearrow$ -1 $\searrow$ 1 $\nearrow$

$\boxed{\begin{array}{l}\text{Incr: } (-\infty,-1)\cup(1,\infty),\ f'(x)>0\\ \text{Decr: } (-1,1),\ f'(x)<0\\ \text{Rel Max: } (-1,4),\ f(x)\text{ incr}\to\text{decr}\\ \text{Rel Min: } (1,0),\ f(x)\text{ decr}\to\text{incr}\end{array}}$

2. $f(x)=\dfrac{x^2-4x+4}{x+2}$

$f'(x)=\dfrac{(2x-4)(x+2)-(x^2-4x+4)}{(x+2)^2}$

$=\dfrac{2x^2-8-x^2+4x-4}{(x+2)^2}=\dfrac{x^2+4x-12}{(x+2)^2}$

$f'(x)=0$

$x^2+4x-12=0$

$(x+6)(x-2)=0$

$x=-6,2$

$f'(x)$ und

$x+2=0$

$x=\cancel{-2}$

NOT in domain

$f'(x)$: $+$ | $-$ | $-$ | $+$

$f(x)$: $\nearrow$ -6 $\searrow$ -2 $\searrow$ 2 $\nearrow$

$\boxed{\begin{array}{l}\text{Incr: } (-\infty,-6)\cup(2,\infty),\ f'(x)>0\\ \text{Decr: } (-6,-2)\cup(-2,2),\ f'(x)<0\\ \text{Rel Max: } (-6,-16),\ f(x)\text{ incr}\to\text{decr}\\ \text{Rel Min: } (2,0),\ f(x)\text{ decr}\to\text{incr}\end{array}}$

3. $f(x)=x^3+3x^2-45x+17$

$f'(x)=3x^2+6x-45$

$f'(x)=0$

$3x^2+6x-45=0$

$x^2+2x-15=0$

$(x+5)(x-3)=0$

$x=-5,3$

~~f'(x) und~~

NOT Poss

$f'(x)$: $+$ | $-$ | $+$

$f(x)$: $\searrow$ -5 $\nearrow$ 3 $\searrow$

$\boxed{\begin{array}{l}\text{Incr: } (-\infty,-5)\cup(3,\infty),\ f'(x)>0\\ \text{Decr: } (-5,3),\ f'(x)<0\\ \text{Rel Max: } (-5,192)\ \ f(x)\text{ incr}\to\text{decr}\\ \text{Rel Min: } (3,-64)\ \ f(x)\text{ decr}\to\text{incr}\end{array}}$

4. $f(x)=\dfrac{1}{2}x+\cos x \qquad (0,2\pi)$

$f'(x)=\frac{1}{2}-\sin x$

$f'(x)=0$

$\frac{1}{2}-\sin x=0$

$\sin x=\frac{1}{2}$

$x=\frac{\pi}{6},\frac{5\pi}{6}$

~~f'(x) und~~

NOT Poss

$f'(x)$: $+$ | $-$ | $+$

$f(x)$: 0 $\nearrow$ $\pi/6$ $\searrow$ $5\pi/6$ $\nearrow$ 2π

$\boxed{\begin{array}{l}\text{Incr: } (0,\pi/6)\cup(5\pi/6,2\pi)\ \ f'(x)>0\\ \text{Decr: } (\pi/6,5\pi/6)\ \ f'(x)<0\\ \text{Rel Max: } (\pi/6,\ \pi/12+\sqrt{3}/2)\ \ f(x)\text{ incr}\to\text{decr}\\ \text{Rel Min: } (5\pi/6,\ 5\pi/12-\sqrt{3}/2)\ \ f(x)\text{ decr}\to\text{incr}\end{array}}$

5. $f(x) = x^4 - 32x + 4$

$f'(x) = 4x^3 - 32$

$f'(x) = 0$

$4x^3 - 32 = 0$

$x^3 - 8 = 0$

$(x-2)\underbrace{(x^2+2x+4)}_{\text{No soln}} = 0$

$x = 2$

~~f'(x) und~~ NOT Possible

$f'(x)$: $-$ | $+$ (at 2)

$f(x)$: ↘ 2 ↗

$\boxed{\begin{array}{l}\text{Incr: } (2,\infty),\ f'(x)>0 \\ \text{Dec: } (-\infty,2),\ f'(x)<0 \\ \text{Rel Max: None} \\ \text{Rel Min: } (2,-44)\ f(x) \text{ decr} \to \text{incr}\end{array}}$

6. $f(x) = (x-1)^{\frac{1}{3}}$

$f'(x) = \dfrac{1}{3(x-1)^{2/3}}$

~~f'(x) = 0~~ NOT Poss.

$f'(x)$ und

$3(x-1)^{2/3} = 0$

$x - 1 = 0$

$x = 1$

$f'(x)$: $+$ | $+$ (at 1)

$f(x)$: ↗ 1 ↗

$\boxed{\begin{array}{l}\text{Incr: } (-\infty,1)\cup(1,\infty)\ f'(x)>0 \\ \text{Decr: None} \\ \text{Rel Max: None} \\ \text{Rel Min: None}\end{array}}$

7. $f(x) = \dfrac{x^2}{x-1}$

$f'(x) = \dfrac{2x(x-1) - x^2}{(x-1)^2}$

$f'(x) = \dfrac{2x^2 - 2x - x^2}{(x-1)^2}$

$f'(x) = \dfrac{x^2-2x}{(x-1)^2} = \dfrac{x(x-2)}{(x-1)^2}$

$f'(x) = 0$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, 2$

$f'(x)$ und

$(x-1)^2 = 0$

~~$x = 1$~~ NOT in domain

$f'(x)$: $+$ | $-$ | $-$ | $+$

$f(x)$: ↗ 0 ↘ 1 ↘ 2 ↗

$\boxed{\begin{array}{l}\text{Incr: } (-\infty,0)\cup(2,\infty),\ f'(x)>0 \\ \text{Decr: } (0,1)\cup(1,\infty),\ f'(x)<0 \\ \text{Rel Max: } (0,0)\ f(x) \text{ incr} \to \text{decr} \\ \text{Rel Min: } (2,4)\ f(x) \text{ decr} \to \text{incr}\end{array}}$

Determine whether the statement is true or false. If false, explain why or give an example that shows it is false.

8. Every nth degree polynomial has $(n-1)$ critical numbers.

false $f(x) = (x-1)^4 \Rightarrow$ degree 4

$f'(x) = 4(x-1)^3$

Only 1 Critical #

9. An nth degree polynomial has at most $(n-1)$ critical numbers.

True!

10. There is a relative maximum or minimum at each critical #.

false $f(x) = x^3$

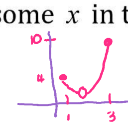
$f'(x) = 3x^2$

$x=0$ crit # but NOT extrema

$f'(x)$: $+$ | $+$

$f(x)$: ↗ 0 ↗

11. The relative maxima of the function f are $f(1) = 4$ and $f(3) = 10$. f must have at least one minimum for some x in the interval (1, 3).

false $\Rightarrow$  No min!

True if $f(x)$ is continuous!