4.5 Review

Sunday, April 29, 2018 8:14 PM

Calculus H – Section 4.5 Quiz Review
Integration by Substitution

Name ___________________________

<u>SHOW ALL WORK ON A SEPARATE SHEET IN YOUR NOTEBOOK!</u>

Evaluate the following indefinite integrals:

1. $\int \frac{3x}{\sqrt{x^2-5}}\,dx = \int \frac{3\cancel{x}}{\sqrt{u}} \cdot \frac{du}{2\cancel{x}}$

$u = x^2-5$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$= \frac{3}{2}\int u^{-1/2}\,du$

$= \frac{3}{2} \cdot 2u^{1/2} + C = \boxed{3\sqrt{x^2-5} + C}$

2. $\int 5x\sqrt{x^2-4}\,dx = \int 5\cancel{x}\sqrt{u} \cdot \frac{du}{2\cancel{x}} = \frac{5}{2}\int u^{1/2}\,du$

$u = x^2-4$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$= \frac{5}{2} \cdot \frac{2}{3} u^{3/2} + C$

$= \boxed{\frac{5}{3}(x^2-4)^{3/2} + C}$

3. $\int \sqrt{\tan x}\sec^2 x\,dx = \int \sqrt{u}\,\cancel{\sec^2 x} \cdot \frac{du}{\cancel{\sec^2 x}} = \int u^{1/2}\,du$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$= \frac{2}{3}u^{3/2} + C$

$= \boxed{\frac{2}{3}\sqrt{\tan^3 x} + C}$

Evaluate the following definite integrals:

8. $\int_{-1}^{2} x(x^2-4)\,dx = \int_{-3}^{0} \cancel{x}\, u \cdot \frac{du}{2\cancel{x}}$

$u = x^2-4$

$\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$

$x=-1,\ u=-3$

$x=2,\ u=0$

$= \frac{1}{2}\int_{-3}^{0} u\,du$

$= \frac{1}{2}\frac{u^2}{2}\Big|_{-3}^{0} = \frac{1}{4}(0^2-(-3)^2)$

$= \boxed{-\frac{9}{4}}$

9. $\int_0^3 \frac{1}{\sqrt{1+x}}\,dx = \int_1^4 \frac{1}{\sqrt{u}}\,du = \int_1^4 u^{-1/2}\,du$

$u = 1+x$

$\frac{du}{dx} = 1 \Rightarrow dx = du$

$x=0,\ u=1$

$x=1,\ u=4$

$= 2u^{1/2}\Big|_1^4 = 2(\sqrt{4}-\sqrt{1})$

$= \boxed{2}$

10. $\int_0^{\pi} \cos\left(\frac{x}{2}\right)dx = \int_0^{\pi/2} \cos(u) \cdot 2\,du$

$u = \frac{x}{2}$

$\frac{du}{dx} = \frac{1}{2}$

$dx = 2\,du$

$x=0,\ u=0$

$x=\pi,\ u=\pi/2$

$= 2\sin u\Big|_0^{\pi/2}$

$= 2(\sin\pi/2 - \sin 0) = \boxed{2}$

4. $\int 4x^3\cos(x^4)\,dx = \int \cancel{4x^3}\cos(u)\,\frac{du}{\cancel{4x^3}}$

$u = x^4$

$\frac{du}{dx} = 4x^3$

$dx = \frac{du}{4x^3}$

$= \sin(u) + C$

$= \boxed{\sin(x^4) + C}$

5. $\int \pi\sin(\pi x)\,dx = \int \cancel{\pi}\sin u \cdot \frac{du}{\cancel{\pi}}$

$u = \pi x$

$\frac{du}{dx} = \pi$

$dx = \frac{du}{\pi}$

$= -\cos u + C$

$= \boxed{-\cos(\pi x) + C}$

6. $\int 5x^2(2x^3+9)^2\,dx = \int 5\cancel{x^2}(u)^2 \cdot \frac{du}{6\cancel{x^2}}$

$u = 2x^3+9$

$\frac{du}{dx} = 6x^2$

$dx = \frac{du}{6x^2}$

$= \frac{5}{6}\int u^2\,du = \frac{5}{6}\cdot\frac{u^3}{3} + C$

$= \boxed{\frac{5}{18}(2x^3+9)^3 + C}$

7. $\int \frac{x}{\sqrt{1+2x^2}}\,dx$

$u = 1+2x^2$

$\frac{du}{dx} = 4x$

$dx = \frac{du}{4x}$

$\int \frac{\cancel{x}}{\sqrt{u}}\cdot\frac{du}{4\cancel{x}} = \frac{1}{4}\int u^{-1/2}\,du$

$= \frac{1}{4}\cdot 2u^{1/2} + C = \boxed{\frac{\sqrt{1+2x^2}}{2} + C}$

11. $\int_0^7 x^3\sqrt{x^4+1}\,dx = \int_1^{2402} \cancel{x^3}\sqrt{u}\cdot\frac{du}{4\cancel{x^3}}$

$u = x^4+1$

$\frac{du}{dx} = 4x^3 \Rightarrow dx = \frac{du}{4x^3}$

$x=0, u=1$

$x=7, u=2402$

$= \frac{1}{4}\int_1^{2402} u^{1/2}\,du = \frac{1}{4}\cdot\frac{2}{3}u^{3/2}\Big|_1^{2402}$

$= \frac{1}{6}\left(2402^{3/2} - 1^{3/2}\right) = \boxed{19620.251}$

12. $\int_0^{\frac{\pi}{2}} \sin\left(\frac{2x}{3}\right)dx = \int_0^{\pi/3} \sin u \cdot \frac{3}{2}du$

$u = \frac{2x}{3}$

$\frac{du}{dx} = \frac{2}{3} \Rightarrow dx = \frac{3}{2}du$

$x=0, u=0$

$x=\pi/2, u=\pi/3$

$= -\frac{3}{2}\cos u\Big|_0^{\pi/3}$

$= -\frac{3}{2}\left(\cos\pi/3 - \cos 0\right)$

$= -\frac{3}{2}\left(\frac{1}{2} - 1\right) = \boxed{\frac{3}{4}}$

13. $\int_0^1 x(x^2+1)^3\,dx$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$x=0, u=1$

$x=1, u=2$

$= \int_1^2 \cancel{x}u^3\cdot\frac{du}{2\cancel{x}} = \frac{1}{2}\int_1^2 u^3\,du$

$= \frac{1}{2}\cdot\frac{u^4}{4}\Big|_1^2 = \frac{1}{8}\left[2^4 - 1^4\right]$

$= \boxed{\frac{15}{8}}$

Find the particular solutions of the differential equations that satisfy the given initial conditions:

1. $\int f'(x)\,dx = \int \sqrt[3]{3x+2}\,dx$; $f(2)=9$

$u = 3x+2$

$\frac{du}{dx} = 3 \Rightarrow dx = \frac{du}{3}$

$f(x) = \int u^{1/3} \cdot \frac{du}{3} = \frac{1}{3} \cdot \frac{3}{4} u^{4/3} + C$

$f(x) = \frac{1}{4}(3x+2)^{4/3} + C$

$f(2) = \frac{1}{4}(8)^{4/3} + C = 9$

$\frac{1}{4}(16) + C = 9$

$4 + C = 9 \Rightarrow C = 5$

$\boxed{f(x) = \frac{1}{4}(3x+2)^{4/3} + 5}$

2. $\int \frac{dy}{dx}\,dx = \int x\sqrt{x^2+5}\,dx$; $y = 12$ if $x = 2$

$u = x^2+5$

$\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$

$y = \int \cancel{x}\sqrt{u} \cdot \frac{du}{2\cancel{x}} = \frac{1}{2}\int u^{1/2}\,du$

$y = \frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C = \frac{1}{3}(x^2+5)^{3/2} + C$

$12 = \frac{1}{3}(9)^{3/2} + C$

$12 = 9 + C \Rightarrow C = 3$

$\boxed{y = \frac{1}{3}(x^2+5)^{3/2} + 3}$

3. $\int f'(x)\,dx = \int \cos 4x\,dx$; $f\left(\frac{3\pi}{8}\right) = \frac{3}{4}$

$u = 4x$

$\frac{du}{dx} = 4 \Rightarrow dx = \frac{du}{4}$

$f(x) = \int \cos u \cdot \frac{du}{4}$

$f(x) = \frac{1}{4}\sin u + C$

$f(x) = \frac{1}{4}\sin 4x + C$

$f\left(\frac{3\pi}{8}\right) = \frac{1}{4}\sin\left(\frac{3\pi}{2}\right) + C = \frac{3}{4}$

$-\frac{1}{4} + C = \frac{3}{4} \Rightarrow C = 1$

$\boxed{f(x) = \frac{1}{4}\sin 4x + 1}$

4. $\int \frac{dy}{dx}\,dx = \int \frac{x}{\sqrt{x^2+9}}\,dx$; $y = 10$ if $x = \sqrt{7}$

$u = x^2+9$

$\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$

$y = \int \frac{\cancel{x}}{\sqrt{u}} \cdot \frac{du}{2\cancel{x}}$

$y = \frac{1}{2}\int u^{-1/2}\,du$

$y = u^{1/2} + C = \sqrt{x^2+9} + C$

$10 = \sqrt{16} + C \Rightarrow C = 6$

$\boxed{y = \sqrt{x^2+9} + 6}$

5. $\int f'(x)\,dx = \int \frac{1}{(3-2x)^2}\,dx$; $f(2) = 5\frac{1}{2}$

$u = 3-2x$

$\frac{du}{dx} = -2$

$dx = \frac{du}{-2}$

$f(x) = \int \frac{1}{u^2} \cdot \frac{du}{-2} = -\frac{1}{2}\int u^{-2}\,du$

$f(x) = \frac{1}{2u} + C$

$f(x) = \frac{1}{2(3-2x)} + C$

$f(2) = \frac{1}{2(-1)} + C = 5\frac{1}{2}$

$-\frac{1}{2} + C = 5\frac{1}{2} \Rightarrow C = 6$

$\boxed{f(x) = \frac{1}{2(3-2x)} + 6}$