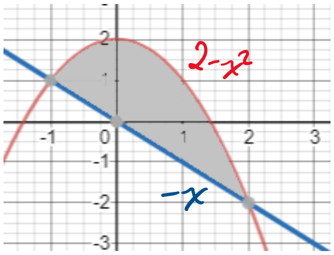
Name_________________________________ Date_______________

Calc I H – Review for 7.1-7.3 Test Period ____________

Find each:

1. The **area** of the region enclosed by the graphs of $y = 2 - x^2$ and the line $y = -x$.

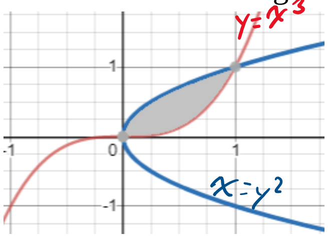


$$A = \int_{-1}^{2} 2 - x^2 - (-x)\,dx$$

$$= \int_{-1}^{2} 2 - x^2 + x\,dx$$

$$\boxed{= 4\tfrac{1}{2}u^2}$$

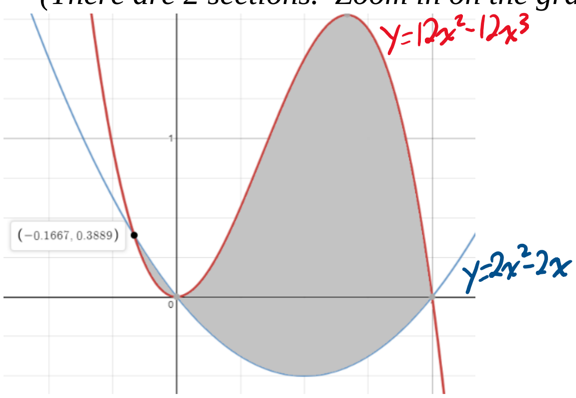
2. The **area** of the region enclosed by the graphs of $y = x^3$ and the line $x = y^2$.



$$A = \int_0^1 \sqrt{x} - x^3\,dx \quad \text{or} \quad \int_0^1 \sqrt[3]{y} - y^2\,dy$$

$$\boxed{= 0.41\overline{6}\,u^2}$$

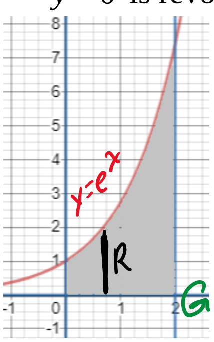
3. The **area** of the region enclosed by the graphs of $y = 12x^2 - 12x^3$ and the line $y = 2x^2 - 2x$.
   (There are 2 sections. Zoom in on the graph to see them)



$$A = \int_{-.1\overline{6}}^{0} 2x^2 - 2x - 12x^2 + 12x^3\,dx + \int_0^1 12x^2 - 12x^3 - 2x^2 + 2x\,dx$$

$$\boxed{\approx 1.343u^2}$$

4. The **volume** of the solid formed when the graph of the region bounded by $y = e^x$, $x = 0$, $x = 2$ and $y = 0$ is revolved about the *x*-axis.

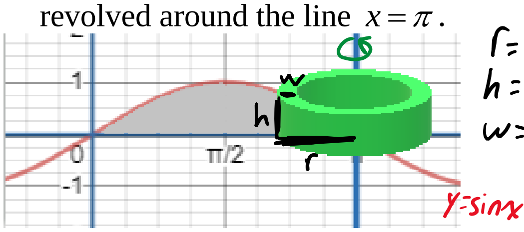


$R = e^x - 0$

$$V = \pi \int_0^2 (e^x)^2\,dx$$

$$\boxed{\approx 84.192u^3}$$

5. The **volume** when the region *R* bounded by the function $y = \sin(x)$, $x = 0$, $x = \pi$ and $y = 0$ is revolved around the line $x = \pi$.

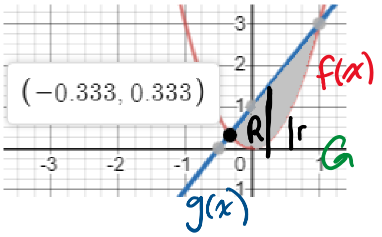


$r = \pi - x$
$h = \sin x$
$w = dx$

$$V = 2\pi \int_0^{\pi} (\pi - x)(\sin x)\,dx$$

$$\boxed{\approx 19.739u^3}$$

6. The **volume** of the solid formed by revolving the region bounded by $f(x)=3x^2$ and $g(x)=2x+1$ about the x-axis.



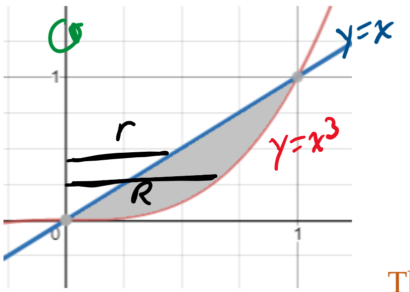
$R = 2x+1-0$

$r = 3x^2-0$

$$V=\pi\int_{-\frac{1}{3}}^{1}(2x+1)^2-(3x^2)^2\,dx$$

$\boxed{\approx 8.440u^3}$

7. The **volume** of the solid formed by revolving the region bounded by the graph of $y=x^3$ and the line $y=x$, between $x=0$ and $x=1$, about the y-axis.



$R=\sqrt[3]{y}-0$

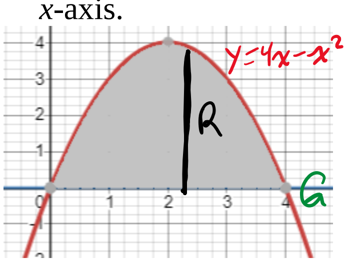
$r=y-0$

$$V=\pi\int_0^1(\sqrt[3]{y})^2-y^2\,dy$$

$\boxed{\approx 0.838u^3}$

This could also be done using the shell method $V=2\pi\int_0^1 x(x-x^3)\,dx$

8. The **volume** of the solid formed by revolving the region bounded by $y=4x-x^2$ and $y=0$ about the x-axis.

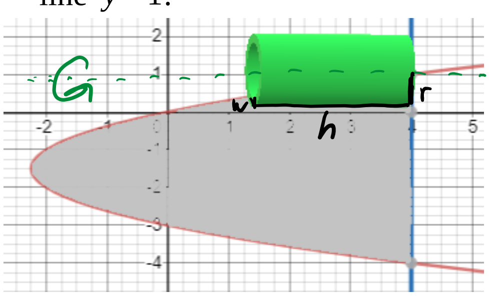


$R=4x-x^2-0$

$$V=\pi\int_0^4(4x-x^2)^2\,dx$$

$\boxed{\approx 107.233u^3}$

9. The **volume** of the solid formed by revolving the region bounded by $x=y^2+3y$ and $x=4$ about the line $y=1$.



$r=1-y$

$h=4-(y^2+3y)$

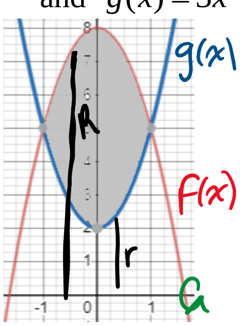
$4-y^2-3y$

$w=dy$

$$V=2\pi\int_{-4}^{1}(1-y)(4-y^2-3y)\,dy$$

$\boxed{\approx 327.249u^3}$

10. The **volume** of the solid formed by revolving the region bounded by the graph of $f(x)=-3x^2+8$ and $g(x)=3x^2+2$ about the x-axis.



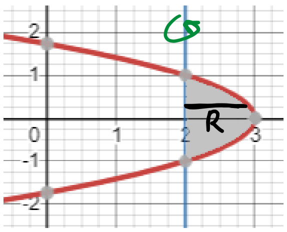
$R=-3x^2+8-0$

$r=3x^2+2-0$

$$V=\pi\int_{-1}^{1}(-3x^2+8)^2-(3x^2+2)^2\,dx$$

$\boxed{\approx 251.327u^3}$

11. The **volume** of the solid formed by revolving the region bounded by the graph of $x = 3 - y^2$ and $x = 2$ about the line $x = 2$.

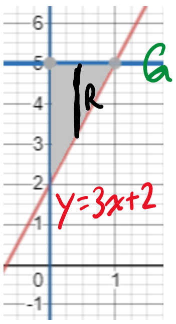


$R = 3 - y^2 - 2$
$= 1 - y^2$

$V = \pi \int_{-1}^{1} (1-y^2)^2 \, dy$

$\boxed{\approx 3.351 \, u^3}$

12. The **volume** of the solid formed by revolving the region bounded by the graph of $y = 3x + 2$, $y = 5$, and $x = 0$ about the line $y = 5$.



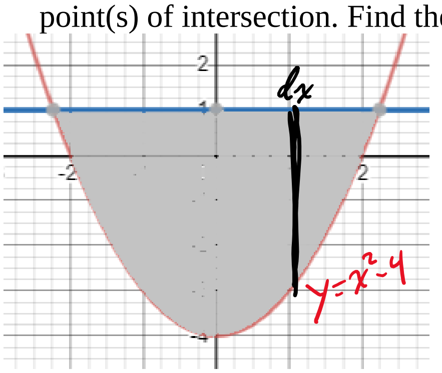
$R = 5 - (3x+2)$
$= 3 - 3x$

$V = \pi \int_0^1 (3-3x)^2 \, dx$

$\boxed{\approx 9.425 \, u^3}$

This could also be done using the shell method $V = 2\pi \int_2^5 (5-y)\left(\frac{y-2}{3}\right) dy$

13. Consider the region R bounded by the functions $y = x^2 - 4$ and $y = 1$. Sketch the region and indicate point(s) of intersection. Find the volume generated when R is revolved about:



a) the line $x = 3$

Shell

$V = 2\pi \int_{-\sqrt{5}}^{\sqrt{5}} (3-x)(1-x^2+4) \, dx$

$\approx 280.993 \, u^3$

b) the line $x = -4$

Shell

$V = 2\pi \int_{-\sqrt{5}}^{\sqrt{5}} (x+4)(5-x^2) \, dx$

$\approx 374.657 \, u^3$

c) the line $y = 2$

Washer

$V = \pi \int_{-\sqrt{5}}^{\sqrt{5}} (6-x^2)^2 - 1^2 \, dx$

$\approx 280.993 \, u^3$

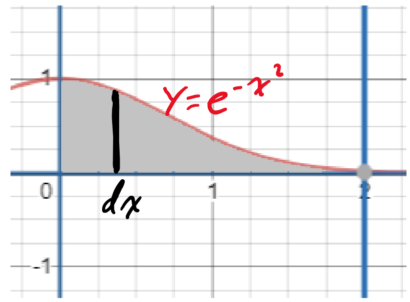
d) the line $y = 1$

Disk

$V = \pi \int_{-\sqrt{5}}^{\sqrt{5}} (5-x^2)^2 \, dx$

$\approx 187.328 \, u^3$

14. Find the volume of the following solids formed by revolving the region bounded by $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = 2$ about the appropriate axis of revolution. Use the integration capabilities of the graphing calculator to find the volume.



a) x-axis

Disk $V = \pi \int_0^2 (e^{-x^2})^2 dx \approx 1.969u^3$

b) $y = -2$

Washer $V = \pi \int_0^2 (e^{-x^2}+2)^2 - 2^2 dx \approx 13.053u^3$

c) $x = -2$

Shell $V = 2\pi \int_0^2 (x+2)(e^{-x^2}) dx \approx 14.169u^3$

d) $x = 3$

Shell $V = 2\pi \int_0^2 (3-x)(e^{-x^2}) dx \approx 13.543u^3$

e) y-axis

Shell $V = 2\pi \int_0^2 x e^{-x^2} dx \approx 3.084u^3$