Calculus H – Integration Concepts Activity **Concept A** **Initials: _______**

1. Find the general solution of the differential equation below and check the result by differentiation.

$$\frac{dF}{dx}=\frac{8}{5}x^{\frac{3}{5}}$$

$$\int dF=\int \frac{8}{5}x^{\frac{3}{5}}dx$$

$$F=\frac{8}{5}\cdot\frac{x^{\frac{8}{5}}}{\frac{8}{5}}+C=x^{\frac{8}{5}}+C$$

or

$$\sqrt[5]{x^8}+C$$

2. Find the indefinite integral and check the result by differentiation. $\int(10-18z)\left(-9z^2+10z\right)^5dz$.

$u=-9z^2+10z$

$\frac{du}{dz}=-18z+10$

$dz=\frac{du}{-18z+10}$

$$=-\int u^5du$$

$$=-\frac{u^6}{6}+C$$

$$=\frac{-(-9z^2+10z)^6}{6}+C$$

3. Solve the differential equation.

$$\frac{dY}{dx}=8x^3-4,\ Y(-1)=4$$

$$\int dY=\int 8x^3-4\,dx$$

$$Y=2x^4-4x+C$$

$4=2(-1)^4-4(-1)+C$

$4=2+4+C$

$-2=C$

$$Y=2x^4-4x-2$$

4. Find the indefinite integral and check the result by differentiation. $\int\frac{4u^2+12u-15}{u^4}du$.

$$=\int 4u^{-2}+12u^{-3}-15u^{-4}\,du$$

$$=\frac{4u^{-1}}{-1}+\frac{12u^{-2}}{-2}+\frac{15u^{-3}}{-3}+C$$

$$=-\frac{4}{u}-\frac{6}{u^2}-\frac{5}{u^3}+C$$

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4. Find the indefinite integral and check the result by differentiation. $\int\frac{4u^2+12u-15}{u^4}du$.

Calculus H – Integration Concepts Activity **Concept B** **Initials: _______**

5. Evaluate the following definite integral. $\int_{-3}^{2} 12t\sqrt{3t^2+5}\,dt$.

$u=3t^2+5$

$dt=\frac{du}{6t}$

$t=2 \Rightarrow u=17$

$t=-3 \Rightarrow u=32$

$= \int_{32}^{17} 12t\, u^{\frac{1}{2}} \frac{du}{6t}$

$= 2\int_{32}^{17} u^{\frac{1}{2}}\, du$

$= 2\left[\frac{2}{3} u^{\frac{3}{2}}\right]_{32}^{17}$

$= \frac{4}{3}\left[17^{\frac{3}{2}} - 32^{\frac{3}{2}}\right] \approx -147.902$

6. Evaluate the following definite integral. $\int_{0}^{1} \sqrt{x^7}\,dx$.

$= \int_0^1 x^{\frac{7}{2}} dx$

$= \left[\frac{2}{9} x^{\frac{9}{2}}\right]_0^1$

$= \frac{2}{9}\left(1^{\frac{9}{2}} - 0^{\frac{9}{2}}\right)$

$= \frac{2}{9}(1)$

$= \frac{2}{9}$

7. Evaluate the following definite integral. $\int_{0}^{7} 10|u-6|\,du$.

Vertex at $u=6$

$-u+6$ when $u<6$

$u-6$ when $u>6$

$= 10\left(\int_0^6 -u+6\,du + \int_6^7 u-6\,du\right)$

$= 10\left[\left[-\frac{u^2}{2}+6u\right]_0^6 + \left[\frac{u^2}{2}-6u\right]_6^7\right]$

$= 10\left(-\frac{36}{2}+36-0+\frac{49}{2}-42-\left(\frac{36}{2}-36\right)\right)$

$= 10\left(-\frac{23}{2}+30\right) = 10\left(\frac{37}{2}\right) = 185$

Calculus H – Integration Concepts Activity **Concept B** **Initials: _______**

5. Evaluate the following definite integral. $\int_{-3}^{2} 12t\sqrt{3t^2+5}\,dt$.

6. Evaluate the following definite integral. $\int_{0}^{1} \sqrt{x^7}\,dx$.

7. Evaluate the following definite integral. $\int_{0}^{7} 10|u-6|\,du$.

Calculus H – Integration Concepts Activity **Concept C** **Initials: _______**

8. Solve the differential equation.

$$\frac{dY}{dx} = xe^{x^2-1}\ ,\ Y(-1) = 4$$

$\int dY = \int x e^{x^2-1} dx$

$u = x^2 - 1$

$dx = \frac{du}{2x}$

$Y = \frac{1}{2}\int e^u du$

$Y = \frac{1}{2}e^u + C$

$Y = \frac{1}{2}e^{x^2-1} + C$

$4 = \frac{1}{2}e^{(-1)^2-1} + C$

$4 = \frac{1}{2}(1) + C$

$4 = \frac{1}{2} + C$

$3\frac{1}{2} = C$

$Y = \frac{1}{2}e^{x^2-1} + 3\frac{1}{2}$

9. Find the indefinite integral. Given $\int \frac{3x^2+1}{3x^3+3x}dx$

$u = 3x^3 + 3x$

$dx = \frac{du}{9x^2+3}$

$= \frac{1}{3}\int \frac{1}{u} du$

$= \frac{1}{3}\ln|u| + C$

$= \frac{1}{3}\ln|3x^3+3x| + C$

10. Find the indefinite integral. Given $\int \frac{e^{\frac{1}{x}}}{x^2}dx$

$u = \frac{1}{x}$

$dx = -x^2 du$

$= -\int \frac{e^u}{x^2} x^2 du$

$= -\int e^u du$

$= -e^u + C$

$= -e^{\frac{1}{x}} + C$

11. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos(x)}{2\sin(x)-1} dx = \int_{-1}^{1} \frac{\cos(x)}{u} \frac{du}{2\cos(x)}$

$u = 2\sin(x) - 1$

$dx = \frac{du}{2\cos(x)}$

$x = \frac{\pi}{2} \rightarrow u = 1$

$x = 0 \rightarrow u = -1$

$= \frac{1}{2}\int_{-1}^{1} \frac{1}{u} du$

$= \frac{1}{2}\Big[\ln|u|\Big]_{-1}^{1}$

$= \frac{1}{2}(\ln(1) - \ln(1))$

$= \frac{1}{2}(0) = 0$

Calculus H – Integration Concepts Activity **Concept C** **Initials: _______**

8. Solve the differential equation.

$$\frac{dY}{dx} = xe^{x^2-1}\ ,\ Y(-1) = 4$$

9. Find the indefinite integral. Given $\int \frac{3x^2+1}{3x^3+3x}dx$

10. Find the indefinite integral. Given $\int \frac{e^{\frac{1}{x}}}{x^2}dx$

11. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos(x)}{2\sin(x)-1} dx$

Calculus H – Integration Concepts Activity **Concept D** **Initials: _______**

12. Find the indefinite integral and check the result by differentiation. $\int (5\sin u + 6\cos u)\,du$.

$= -5\cos u + 6\sin u + C$

13. Evaluate the following definite integral.

$\int_0^{7\pi} (5\sin u + 4\cos u)\,du$.

$7\pi = \pi$ on the unit circle

$= \left[-5\cos u + 4\sin u\right]_0^{7\pi}$

$= -5\cos(7\pi) + 4\sin(7\pi) - (-5\cos(0) + 4\sin(0))$

$= -5(-1) + 4(0) - (-5(1) + 4(0))$

$= 5 + 5$

$= 10$

14. Evaluate the following definite integral.

$\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \csc(2x)\cot(2x)\,dx$. $= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \csc(u)\cot(u)\,du$

$u = 2x$

$dx = \frac{du}{2}$

$x = \frac{\pi}{4} \rightarrow u = \frac{\pi}{2}$

$x = \frac{\pi}{12} \rightarrow u = \frac{\pi}{6}$

$= -\frac{1}{2}\left[\csc(u)\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= -\frac{1}{2}(1 - 2)$

$= \frac{1}{2}$

15. Evaluate the following indefinite integral.

$\int \frac{\sec(x)\tan(x)}{1+\sec(x)}dx$ $= \int \frac{1}{u}\,du$

$u = 1 + \sec(x)$

$dx = \frac{du}{\sec(x)\tan(x)}$

$= \ln|u| + C$

$= \ln|1+\sec(x)| + C$

Calculus H – Integration Concepts Activity **Concept D** **Initials: _______**

12. Find the indefinite integral and check the result by differentiation. $\int (5\sin u + 6\cos u)\,du$.

13. Evaluate the following definite integral.

$\int_0^{7\pi} (5\sin u + 4\cos u)\,du$.

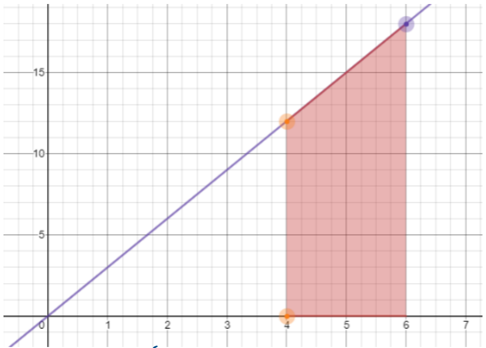
14. Evaluate the following definite integral.

$\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \csc(2x)\cot(2x)\,dx$.

15. Evaluate the following indefinite integral.

$\int \frac{\sec(x)\tan(x)}{1+\sec(x)}dx$

16. Sketch the region and evaluate the following definite integral. $\int_4^6 3z\,dz$.



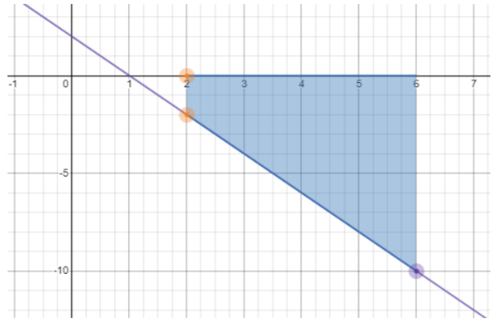
$\left[\frac{3}{2}z^2\right]_4^6$

$\frac{3}{2}(36)-\frac{3}{2}(16)$

$54-24$

30

17. Sketch the region and evaluate the following definite integral. $\int_2^6 (-2u+2)\,du$.



$\left[-u^2+2u\right]_2^6$

$-36+2(6)-(-4+2(2))$

$-24-(0)$

-24

18. Find the area under the curve $y = x^2 - 2$ on the interval $[3,5]$.

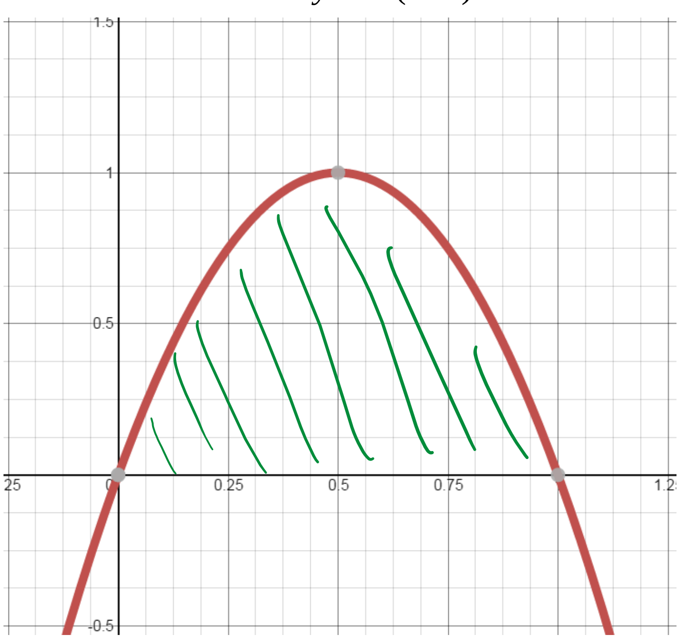
$\int_3^5 x^2-2\,dx = \left[\frac{x^3}{3}-2x\right]_3^5$

$= \frac{5^3}{3}-2(5)-\left(\frac{3^3}{3}-2(3)\right)$

$= \frac{125}{3}-10-9+6$

$= \frac{125}{3}-13 = \frac{86}{3}$

19. Determine the area of $y = 4x(1-x)$ that lies above the x-axis.



$\int_0^1 4x-4x^2\,dx$

$=\left[2x^2-\frac{4}{3}x^3\right]_0^1$

$= 2(1)^2-\frac{4}{3}(1)^3-0$

$= 2-\frac{4}{3}$

$= \frac{2}{3}$