Open Ended practice:

1. Rewrite each Riemann Sum as a Definite Integral.

a. $\lim_{n\to\infty}\sum_{i=1}^{n}\left[\sqrt{4+\frac{16}{n}i}\left(\frac{16}{n}\right)\right]$ lower bound $\Delta x = \frac{16}{n} = \frac{b-a}{n}$ $f(x)=\sqrt{x}$ $= \boxed{\int_4^{20}\sqrt{x}\,dx}$	b. $\lim_{n\to\infty}\sum_{i=1}^{n}\left[\frac{1}{2+\frac{1}{n}i}\left(\frac{1}{n}\right)\right]$ lower bound $\Delta x = \frac{1}{n} = \frac{b-a}{n}$ $f(x)=\frac{1}{x}$ $= \boxed{\int_2^{3}\frac{1}{x}\,dx}$
c. $\lim_{n\to\infty}\sum_{i=1}^{n}\left[e^{\frac{1}{n}i}\left(\frac{1}{n}\right)\right]$ lower bound $=0$ $\Delta x = \frac{1}{n} = \frac{b-a}{n}$ $f(x)=e^x$ $= \boxed{\int_0^{1}e^x\,dx}$	d. $\lim_{n\to\infty}\sum_{i=1}^{n}\left[\cos\left(\frac{2\pi}{n}i\right)\left(\frac{2\pi}{n}\right)\right]$ lower bound $=0$ $\Delta x = \frac{2\pi}{n}$ $f(x)=\cos x$ $= \boxed{\int_0^{2\pi}\cos x\,dx}$

2. Rewrite each Definite Integral as a Riemann Sum.

a. $\int_1^9 \ln x\,dx$ $\Delta x = \frac{9-1}{n} = \frac{8}{n}$ $= \boxed{\lim_{n\to\infty}\sum_{i=1}^{n}\ln\left(1+\frac{8}{n}i\right)\left(\frac{8}{n}\right)}$	b. $\int_{\pi/2}^{3\pi/2}\sin x\,dx$ $\Delta x = \frac{\frac{3\pi}{2}-\frac{\pi}{2}}{n} = \frac{\pi}{n}$ $= \boxed{\lim_{n\to\infty}\sum_{i=1}^{n}\sin\left(\frac{\pi}{2}+\frac{\pi}{n}i\right)\left(\frac{\pi}{n}\right)}$
c. $\int_3^9\left(x^2+2\right)dx$ $\Delta x = \frac{9-3}{n} = \frac{6}{n}$ $= \boxed{\lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(3+\frac{6}{n}i\right)^2+2\right]\left(\frac{6}{n}\right)}$	d. $\int_1^3\lvert x\rvert\,dx$ $\Delta x = \frac{3-1}{n} = \frac{2}{n}$ $= \boxed{\lim_{n\to\infty}\sum_{i=1}^{n}\left\lvert 1+\frac{2}{n}i\right\rvert\left(\frac{2}{n}\right)}$

Multiple Choice:

3. $\int_0^5 \left(3-x^2\right)dx =$ $\Delta x = \frac{5}{n}$

   a. $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left[\left(\frac{5}{n}i\right)^2\left(\frac{5}{n}\right)\right]$

   b. $\boxed{\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left[\left(3-\left(\frac{5}{n}i\right)^2\right)\left(\frac{5}{n}\right)\right]}$

   c. $\lim\limits_{n\to\infty} \sum\limits_{i=0}^{n-1} \left[\left(\frac{5}{n}i\right)^2\left(\frac{5}{n}\right)\right]$

   d. $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left[\left(3-\left(\frac{25}{n}i\right)^2\right)\left(\frac{5}{n}\right)\right]$

4. $\lim\limits_{n\to\infty} \frac{1}{n}\left[2\left(\frac{1}{n}\right)^{2/3}+2\left(\frac{2}{n}\right)^{2/3}+2\left(\frac{3}{n}\right)^{2/3}+\ldots+2\left(\frac{n}{n}\right)^{2/3}\right]=$

   $\Delta x = \frac{1}{n}$ lower bound $=0$ $f(x) = 2x^{2/3}$

   a. $2\int_0^1 \left(\frac{1}{x}\right)^{2/3} dx$ b. $\int_0^2 x^{2/3}dx$ c. $\boxed{\int_0^1 2x^{2/3}dx}$ d. $\frac{2}{n^{2/3}}\int_0^1 dx$

5. The expression $\frac{2}{25}\left[\sqrt{\frac{2}{25}}+\sqrt{\frac{4}{25}}+\sqrt{\frac{6}{25}}+\ldots+\sqrt{\frac{48}{25}}+\sqrt{\frac{50}{25}}\right]$ is a Riemann approximation for

   $\Delta x = \frac{2}{25} = \frac{b-a}{n}$ $b-a=2$ $n=25$

   lower bound $=0$

   a. $\frac{1}{25}\int_0^2 \sqrt{x}\,dx$ b. $\boxed{\int_0^2 \sqrt{x}\,dx}$ c. $\int_0^{50} \sqrt{x}\,dx$ d. $\int_0^1 \sqrt{2x}\,dx$

   $f(x) = \sqrt{x}$

6. If n is a positive integer, then $\lim\limits_{n\to\infty} \frac{1}{n}\left[\frac{1}{1+(1/n)}+\frac{1}{1+(2/n)}+\frac{1}{1+(3/n)}+\ldots+\frac{1}{1+(n/n)}\right]$ can be expressed as

   $\Delta x = \frac{1}{n} = \frac{b-a}{n}$ lower bound $f(x) = \frac{1}{x}$

   a. $\int_1^2 \frac{1}{x+1}dx$ b. $\int_1^2 \frac{2}{x+1}dx$ c. $\int_0^1 \frac{1}{x}dx$ d. $\boxed{\int_1^2 \frac{1}{x}dx}$

7. If n is a positive integer, then $\lim\limits_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+\ldots+\left(\frac{n-1}{n}\right)^2\right]$ can be expressed as

   $\Delta x = \frac{1}{n} = \frac{b-a}{n}$ lower bound $=0$

   $f(x) = x^2$

   a. $\int_0^1 \frac{1}{x^2}dx$ b. $\boxed{\int_0^1 x^2\,dx}$ c. $\int_0^1 \frac{2}{x^2}dx$ d. $\int_0^2 x^2\,dx$

8. The expression $\frac{4}{50}\left[e^{4/50}+e^{8/50}+\ldots+e^{196/50}+e^{200/50}\right]$ is a Riemann approximation for

   $\Delta x = \frac{4}{50} = \frac{b-a}{a}$ lower bound $=0$ $f(x) = e^x$

   $n=50$

   a. $\boxed{\int_0^4 e^x\,dx}$ b. $\frac{1}{50}\int_0^4 e^x\,dx$ c. $\int_4^{50} e^x\,dx$ d. $\int_0^1 e^{4x}\,dx$