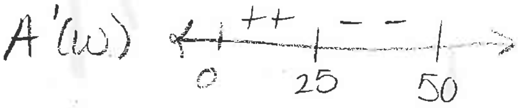
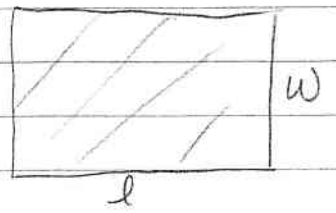


∴ A'(25) = max area

or A'(w) = -2 < 0, ∴ A(25) max

3.7 Classwork

1.



$A = wl$

$P = 2l + 2w$

$100 = 2l + 2w$

$50 = l + w$

$l = 50 - w$

feasible domain

$0 < w < 50$

$A = w(50-w) = 50w - w^2$

$A' = 50 - 2w = 0$

$2w = 50$

$w = 25$

Since $A'(25) = 0$,

$A'' = -2 < 0$, ∴ $w = 25$ max

∴ max area = 25'×25' fenc

= 625 ft^2

2. $V = (16-2x)(30-2x)x$

$V = (480 - 32x - 60x + 4x^2)x$

$V = 480x - 92x^2 + 4x^3$

$V' = 480 - 184x + 12x^2 = 0$

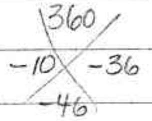
$3x^2 - 46x + 120 = 0$

$3x^2 - 36x - 10x + 120 = 0$

$3x(x-12) - 10(x-12) = 0$

$(3x-10)(x-12) = 0$

$x = 10/3$ or ~~$x = 12$~~ Not feasible

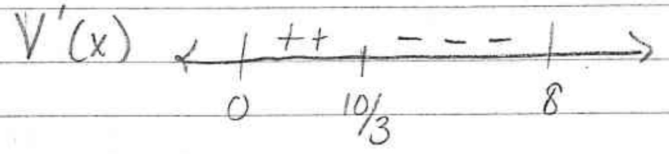


Feasible domain

$16 - 2x > 0$

$16 > 2x$

$0 < x < 8$



∴ V'(x) max at

$x = 10/3$

OR

$V''(x) = 6x - 46$

$V''(10/3) = 6\left(\frac{10}{3}\right) - 46 < 0$

Since $V'(10/3) = 0$ & $V''(10/3) <$

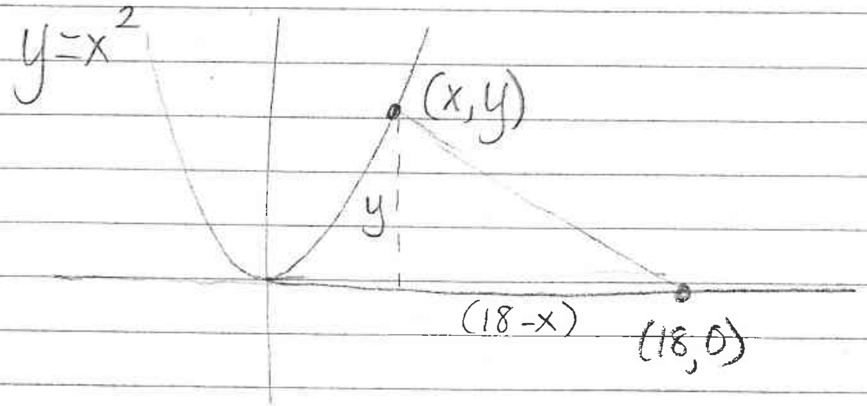
∴ V(x) concave down &

x = 10/3 is a max!

Mav Volume $= 480\left(10/3\right) - 92\left(10/3\right)^2 + 4\left(\frac{10}{3}\right)^3 = 725.93$in

Corners should be 10/3 in X 10/3 in.

3. $y=x^2$



$d^2 = (18-x)^2 + y^2 = (18-x^2) + (x^2)^2$

$d^2 = (18-x)^2 + x^4 \Rightarrow$ d is smallest
$d_1 = \sqrt{(18-x)^2 + x^4}$ when $f(x) = (18-x)^2 + x^4$ smallest

$f(x) = (18-x)^2 + x^4$
$f'(x) = -2(18-x) + 4x^3 = 0$
$4x^3 + 2x - 36 = 0$
$2x^3 + x - 18 = 0$
$x = 2$
$f''(x) = 12x^2 + 2$
Since $f'(2)=0$, $f''(2) > 0$ ∴ $f(x)$ min. when $x=2$
(concave up)

min distance occurs at $(2,4)$

4. $C(x) = 500,000 + 80x + .003x^2$

Profit = $200x - C(x) = 200x - 500,000 - 80x - .003x^2$
$P(x) = -.003x^2 + 120x - 500,000$

$P'(x) = -.006x + 120 = 0$ feasible domain
$.006x = 120$ $0 < x \leq 30,000$
Since $P'(20,000) = 0$, $x = 20,000$ units
$P''(x) = -.006 < 0$ ∴ at $x = 20,000$, Profit is max
Concave down everywhere

3.7 day Cont

5.

x = # increases

	original	new
price	$60	$60+2x$
# buyers	40	$40-x$

$R(x) = (60+2x)(40-x) = 2400+20x-2x^2$

$R'(x) = 20-4x = 0$

critpt $x=5$

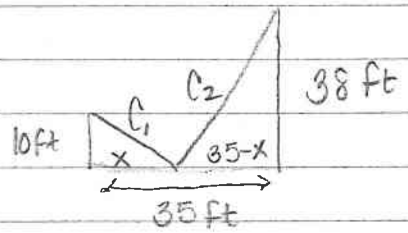
$R''(x) = -4$ ∴ function concave down

$x=5$ is maximum

To maximize revenue, charge $70 per bag, 35 buyers

Rev = $2450

6.



Rope length $= C_1 + C_2$

$R = \sqrt{10^2+x^2} + \sqrt{38^2+(35-x)^2}$

R(x) minimized when C_1 & C_2 are minimal

$f(x) = C_1 + C_2$

$f(x) = 10^2 + x^2 + 38^2 + (35-x)^2$

$f'(x) = 2x - 2(35-x)$

$= 4x - 70 = 0$

$x = \frac{35}{2}$

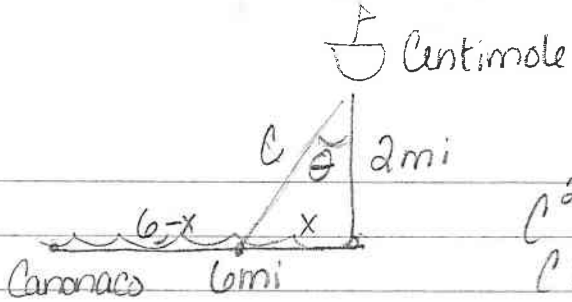
Since $f'(\frac{35}{2}) = 0$,

$f''(x) = 4 > 0$ ∴ Concave up

$x = \frac{35}{2}$ ft.

∴ Stake should be midpoint between poles to minimize the rope length.

7.



$$c^2 = 4 + x^2$$
$$c = \sqrt{4+x^2}$$

$D = Rt \Rightarrow t = \frac{D}{R}$

feasible domain
$0 < x < 6$

$$T_{time} = \frac{\sqrt{4+x^2}}{2} + \frac{6-x}{5}$$

$$T' = \frac{\cancel{2}x}{\cancel{2}\cdot 2\sqrt{4+x^2}} - \frac{1}{5} = 0$$

$$(5x = 2\sqrt{4+x^2})^2$$
$$25x^2 = 4(4+x^2)$$
$$21x^2 = 16$$
$$x^2 = 16/21$$
$$x = \pm 4/\sqrt{21}$$

$$\begin{array}{c|ccc} T' & & - & + \\ \hline T & 0 & \searrow 4/\sqrt{21} & \nearrow \end{array}$$

$\boxed{x = 4/\sqrt{21}}$ min, $T' < 0 \rightarrow T' > 0$

$$\tan\theta = \frac{x}{2} \qquad \theta = \tan^{-1}\left(\frac{2}{\sqrt{21}}\right) \approx 23.58°$$

9. $V = \pi r^2 h$ $h = y$ $x = 2\pi r$
$r = \frac{x}{2\pi}$

$$V = \pi\left(\frac{x}{2\pi}\right)^2 \cdot y$$

$$V = \frac{x^2}{4\pi} \cdot y \qquad 30 = 2x + 2y$$

$$y = \frac{30-2x}{2} = 15 - x$$

$$V(x) = \frac{1}{4\pi} x^2(15-x) = \frac{15x^2 - x^3}{4\pi}$$

$$V'(x) = \frac{30x - 3x^2}{4\pi} = 0$$

$$3x(10-x) = 0$$

$$x = \cancel{0}, 10$$

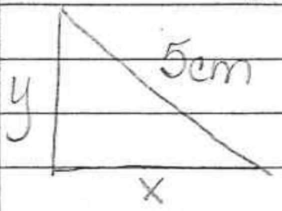
$$V''(x) = 30 - 6x$$

Since $V'(10) = 0$, $V''(10) < 0$, $\therefore x = 10^{cm}$ max!

$y = 15 - x$
$y = 5$cm.

$\boxed{\text{Choice b}}$

3.7 cont

10. 

$A = \frac{xy}{2}$

$25 = x^2 + y^2$

$y = \sqrt{25 - x^2}$

$A = \frac{x\sqrt{25-x^2}}{2}$

Feasible domain:
$0 < X < 5$

$A'(x) = \frac{\sqrt{25-x^2}}{2} + \frac{x \cdot -2x}{4\sqrt{25-x^2}}$

$= \frac{\sqrt{25-x^2}}{2} - \frac{x^2}{2\sqrt{25-x^2}} = 0$

$\frac{\sqrt{25-x^2}}{\cancel{2}} = \frac{x^2}{\cancel{2}\sqrt{25-x^2}}$

$x^2 = 25 - x^2$

$2x^2 = 25$

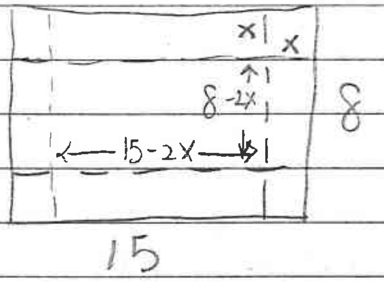
$\sqrt{x^2} = \pm\sqrt{\frac{25}{2}}$

$x \neq -5/\sqrt{2}$

$\boxed{X = \frac{5}{\sqrt{2}} \text{ cm}}$

$A'(x)$ (+ | −)
$A(x)$ 0 ↗ $5/\sqrt{2}$ ↘ 5

max since
$A'(x) > 0 \rightarrow A'(x) <$

11. 

$V = x(8-2x)(15-2x)$

feasible dom
$0 < x < 4$

$V(x) = 120x - 46x^2 + 4x^3$

$V'(x) = 120 - 92x + 12x^2 = 0$

$3x^2 - 23x + 30 = 0$

$(3x - 5)(x - 6) = 0$

$x = 5/3$ $\cancel{x =} 6$ NOT in feasible domain

$V''(x) = -92 + 24x$

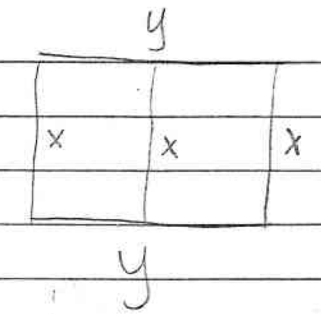
Since $V'(\frac{5}{3}) = 0$, $V''(5/3) = -92 + 40 < 0$

∴ V concave down at $x = 5/3$ in

and $x = \frac{5}{3}$ in max

∴ Box is $\frac{5}{3}$ in X $11\frac{2}{3}$ in X $4\frac{2}{3}$ in

12.



Fence $= 3x + 2y$

$A = 216 = xy$

$y = \frac{216}{x}$

$F(x) = 3x + 2\frac{(216)}{x} = 3x + 432x^{-1}$

feasible domain

$0 < x < 216$

$F'(x) = 3 - \frac{432}{x^2} = 0$

$3x^2 = 432$

$x^2 = 144$

$x = \pm 12 \quad x \neq -12$ NOT in domain

$F''(x) = \frac{864}{x^3}$

$x = 12\text{ m}$

Since $F'(12) = 0$, $F''(12) > 0$ ∴ $F(x)$ concave up when $x = 12$ and $x = 12\text{m}$ is minimum. $y = \frac{216}{12} = 18$

∴ $\boxed{\begin{array}{l}\text{Pea Patsch should be } 12m \times 18m \\ \text{Fence} = 3(12) + 2(18) = 72m \text{ in length}\end{array}}$

13. $S(t) = -16t^2 + 96t + 112$, $t \geq 0$

a) $V(t) = S'(t) = -32t + 96$

$V(0) = \boxed{96\ ft/sec}$

b) $S'(t) = 0$

$-32t + 96 = 0$

$\boxed{t = 3\text{ sec}}$

$\boxed{S(3) = 256\ ft}$

Since $S'(3) = 0$,

$S''(t) = -32 < 0$ Concavedown

∴ critical pt, $t = 3$ is max

c) $\frac{-16t^2 + 96t + 112}{-16} = \frac{0}{-16}$

$t^2 - 6t - 7 = 0$

$(t - 7)(t + 1) = 0$

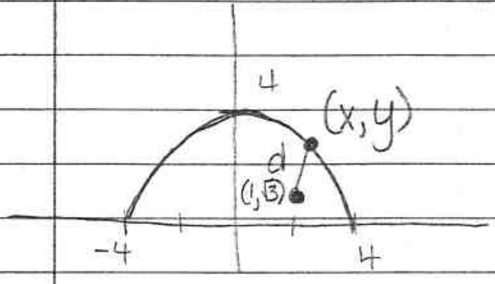
$t = 7$, $\cancel{-1}$ NOT in domain

$\boxed{V(7) = -128\ \frac{ft}{sec}}$

3.7 cont.

14. $y = \sqrt{16 - x^2}$, $(1, \sqrt{3})$

feasible dom: $-4 < x < 4$



$d = \sqrt{(1-x)^2 + (\sqrt{3} - y)^2}$,

$d = \sqrt{(1-x)^2 + (\sqrt{3} - \sqrt{16-x^2})^2}$

d will be minimized when
$f = (1-x)^2 + (\sqrt{3} + \sqrt{16-x^2})^2$ is min.

$$f'(x) = -2(1-x) + 2(\sqrt{3} - \sqrt{16-x^2}) \cdot \frac{-1}{\cancel{2}\sqrt{16-x^2}} \cdot -\cancel{2}x$$

$$= -2 + 2x + \frac{2\sqrt{3}\,x - 2x\sqrt{16-x^2}}{\sqrt{16-x^2}}$$

$$= \frac{-2\sqrt{16-x^2} + \cancel{2x\sqrt{16-x^2}} + 2\sqrt{3}x - \cancel{2x\sqrt{16-x^2}}}{\sqrt{16-x^2}}$$

$$= \frac{-2\sqrt{16-x^2} + 2\sqrt{3}\,x}{\sqrt{16-x^2}} = 0$$

$2\sqrt{16-x^2} = 2\sqrt{3}x$
$\sqrt{16-x^2} = \sqrt{3}x$
$16 - x^2 = 3x^2$
$16 = 4x^2$
$x^2 = 4$
$x = \pm 2$

$f'(x)$ (− | − | +)
$f(x)$ −4 −2 2 4

$x = 2$ min since $f'(x) > 0 \rightarrow f'(x) = 0$

$(2, 2\sqrt{3})$ on curve $y = \sqrt{16-x^2}$ is closest to $(1, \sqrt{3})$.

$d = \sqrt{(1-2)^2 + (\sqrt{3} - 2\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = \boxed{2}$

15. $r(x)=6x$. $C(x)=x^3-6x^2+15x$

$P(x)= 6x-(x^3-6x^2+15x)$ $x\geq 0$

$= -x^3+6x^2-9x$

$P'(x) = -3x^2+12x-9 = 0$

$-3(x^2-4x+3)=0$

$-3(x-3)(x-1)=0$

$x=1,3$

$P''(x)=-6x+12$ Since $P'(3)=0$,

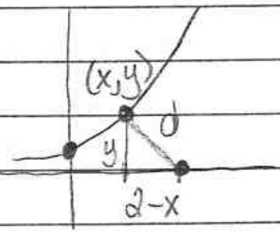
$P''(1)>0$ Concave up $P''(3)<0$ Concave dow

min max

$\therefore x=3$ max $P(3)=0$

At max point → No Profit, break even!

16. $f(x)=e^{\frac{x}{2}}$



$d=\sqrt{(2-x)^2+y^2}$

$d=\sqrt{(2-x)^2+e^x}$

$f(x)=(2-x)^2+e^x$

d will be minimized when f is min

$f'(x)=-2(2-x)+e^x$

$=-4+2x+e^x=0$

On GC → $x=0.841$

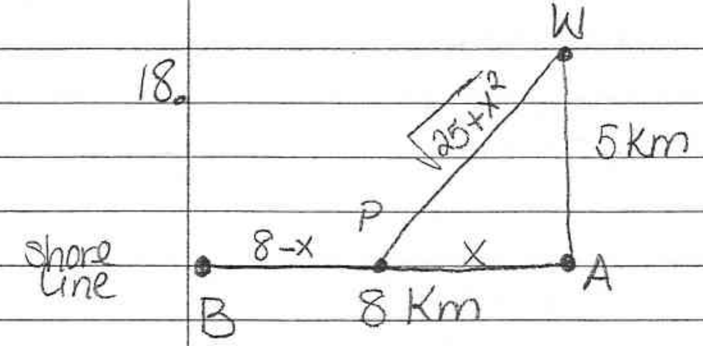
Since $f'(.841)=0$, $f''(x)=2+e^x>0$

Concave up! ∴ $x=0.841$ is min

$\boxed{(0.841,\ 1.523)}$

3.7 Cont.

18.

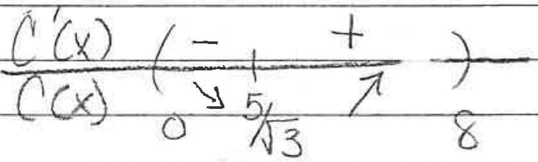


$C = \sqrt{25+x^2}\,(1{,}000{,}000) + (8-x)(500{,}00$

feasible domain
$0 < x < 8$

$C' = \frac{1{,}000{,}000}{\cancel{2}\sqrt{25+x^2}} \cdot \cancel{2}x - 500{,}000$

$C' = 0 \Rightarrow \frac{1{,}000{,}000x}{\sqrt{25+x^2}} = 500{,}000$



$2x = \sqrt{25+x^2}$
$4x^2 = 25 + x^2$
$3x^2 = 25$
$x = \pm 5/\sqrt{3}$
$x = -5/\sqrt{3}$ NOT in domain

Since $C'(x) < 0 \rightarrow C'(x) > 0$
$x = \frac{5}{\sqrt{3}}$ location of min.

$\boxed{\frac{5}{\sqrt{3}} \text{ Km from Point A}}$

19. At min. booking
\$200 / person
50 people

Add'l. booking
\$(200 − 2x)/person
(50 + x) people

x = # people
F.D. $0 \le x \le 80$

$\text{Cost} = 6000 + 32(50+x)$

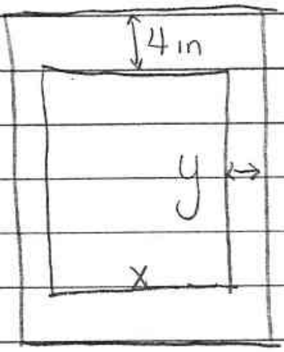
$P(x) = \overset{R(x)}{(200-2x)(50+x)} - \overset{C(x)}{(6000+32(50+x))}$

$P'(x) = -2(50+x) + 200 - 2x - 32$
$= -100 - 2x + 200 - 2x - 32$
$= -4x + 68 = 0$

$\boxed{x = 17 \text{ people}}$

Since $P'(17) = 0$, $P''(x) = -4 < 0$
Concave down, so $x = 17$ max

20.



Overall dimensions

$xy = 50 \text{ in}^2$ $\quad y = \frac{50}{x}$

$A = (x+4)(y+8)$

$A(x) = (x+4)\left(\frac{50}{x} + 8\right)$

FD $0 < x < 5\sqrt{2}$

$A'(x) = \frac{50}{x} + 8 - \frac{50}{x^2}(x+4) = 0$

$\cancel{\frac{50}{x}} + 8 - \cancel{\frac{50}{x}} - \frac{200}{x^2} = 0$

$8 = \frac{200}{x^2}$

$8x^2 = 200$

$x^2 = 25$

$x = \pm 5$ $\quad x = -5$

NOT in domain

$A''(x) = \frac{-100}{x^3}$

Since $A'(5) = 0$, $A''(5) < 0$ $\therefore$ $A(x)$ Concave down and $\boxed{x=5 \text{ location of max}}$

$\therefore$ $x = 5$
$y = 10$

Overall dimension 9in X 18in