Do Now:

1. Let $f(x)=x^7+x+1$ and show that f^{-1} exists (*but do not attempt to find it*).

$f'(x)=x^6+1>0$ for all x. Since $f'(x)>0$, function is always increasing $\Rightarrow$ MONOTONIC & one to one. Therefore, $f^{-1}(x)$ exists.

2. Use graphical reasoning to determine if the following statements are true or false. If false, modify the statement to make it correct.

   a. If $f(x)$ is increasing, then $f^{-1}(x)$ is increasing. true

   b. If $f(x)$ is decreasing, then $f^{-1}(x)$ is decreasing. true

   c. If $f(x)$ is concave up, then $f^{-1}(x)$ is concave ~~up~~ down. false

   d. Linear functions $f(x)=ax+b$ $(a\neq 0)$ are always one-to-one. true

   e. Quadratic functions $f(x)=ax^2+bx+c$ $(a\neq 0)$ are ~~always~~ never one-to-one. false

   f. $\sin(x)$ is not one-to-one. true

Class Work:

1. If $f(x)=x^3-x-6$ for all real numbers x, and if g is the inverse function of f, then $f'(g(0))g'(0)=$

$\rightarrow$ Then $f(g(x))=x$

a. 0 $\quad$ $\boxed{\text{b. } 1}$ $\quad$ c. -1 $\quad$ d. -6 $\quad$ e. $\sqrt[3]{6}$

$f'(g(x))\cdot g'(x)=1$

So $f'(g(0))\cdot g'(0)=1$

2. Given $f(x)=x^5+2x-1$, evaluate $(f^{-1})'(2)$ without calculating the inverse function.

$f(x)=x^5+2x-1$		$f^{-1}(x)$
$(C,2)$	$\Leftarrow$	$(2,C)$

$2=C^5+2C-1$

$0=C^5+2C-3$

G.C. $\quad C=1 \Longrightarrow$ $f'(x)=5x^4+2$

$f'(1)=7$

$\therefore \boxed{(f^{-1})'(2)=\frac{1}{7}}$

3. Directions: Find $(f^{-1})'(x)$ for the function f at the given x value.

a. $f(x)=x^3+2;\quad x=-1$

$-1=x^3+2$

$x^3+3=0$

$x=-\sqrt[3]{3}$

$(f^{-1})'(x)=\dfrac{1}{f'(-\sqrt[3]{3})}=\dfrac{1}{3\sqrt[3]{9}}\approx .160$

$f'(x)=3x^2$

b. $f(x)=x\sqrt{x-3};\quad x=4$

$4=x\sqrt{x-3}$

$x\sqrt{x-3}-4=0$

$x=4$

$(f^{-1})'(x)=\dfrac{1}{f'(4)}=\dfrac{1}{3}$

$f'(x)=\dfrac{x}{2\sqrt{x-3}}+\sqrt{x-3}$

c. $f(x)=\sqrt{x+1}$; $x=2$

$$\boxed{\begin{gathered} 2=\sqrt{x+1} \\ \sqrt{x+1}-2=0 \\ x=3 \\ (f^{-1})'(x)=\frac{1}{f'(3)}=4 \\ f'(x)=\frac{1}{2\sqrt{x+1}} \end{gathered}}$$

d. $f(x)=x^3+x+2$; $x=-3$

$$\boxed{\begin{gathered} -3=x^3+x+2 \\ x^3+x+5=0 \\ x\approx -1.51598 \\ (f^{-1})'(x)=\frac{1}{f'(-1.51598)}\approx \boxed{.127} \\ f'(x)=3x^2+1 \end{gathered}}$$

e. $f(x)=x^3-3x^2+4x+6$; $x=8$

$$\boxed{\begin{gathered} 8=x^3-3x^2+4x+6 \\ x^3-3x^2+4x-2=0 \\ x=1 \\ (f^{-1})'(x)=\frac{1}{f'(1)}=1 \\ f'(x)=3x^2-6x+4 \end{gathered}}$$

f. $f(x)=x+\cos(x)$; $x=1$

$$\boxed{\begin{gathered} 1=x+\cos(x) \\ x+\cos(x)-1=0 \\ x=0 \\ (f^{-1})'(x)=\frac{1}{f'(0)}=1 \\ f'(x)=1-\sin(x) \end{gathered}}$$

4. Show that $f(x)=\frac{1}{1+x}$ and $g(x)=\frac{1-x}{x}$ are inverses. Then compute $g'(x)$ directly and verify that $g'(x)=\frac{1}{f'(g(x))}$.

$f(g(x))=f\left(\frac{1-x}{x}\right)=\frac{(1)x}{\left(1+\frac{1-x}{x}\right)x}=\frac{x}{\cancel{x}+1-\cancel{x}}=x$ ✓

$g(f(x))=g\left(\frac{1}{1+x}\right)=\frac{\left(1-\frac{1}{1+x}\right)(1+x)}{\left(\frac{1}{1+x}\right)(1+x)}=\frac{\cancel{1}+x-\cancel{1}}{1}=x$ ✓

$g'(x)=\frac{-x-(1-x)}{x^2}=\frac{-x-1+x}{x^2}=\boxed{\frac{-1}{x^2}}$

$f'(x)=\frac{-1}{(1+x)^2}$

$f'(g(x))=\frac{-1}{\left(1+\frac{1-x}{x}\right)^2}=\frac{-1}{\left(\frac{x+1-x}{x}\right)^2}=\boxed{\frac{-1}{x^2}}$

$g'(x)=-\frac{1}{x^2}=f'(g(x))$ ✓

5. Find the inverse $g(x)$ of $f(x)=\sqrt{x^2+9}$ with $x\ge 0$. Calculate $g'(5)$ using the theorem learned in class and by finding and differentiating the inverse.

$y=\sqrt{x^2+9}$

$x=\sqrt{y^2+9}$

$x^2=y^2+9$

$y^2=x^2-9$

$y=\sqrt{x^2-9}$

$g(x)=\sqrt{x^2-9}$

$g'(x)=\frac{2x}{2\sqrt{x^2-9}}$

$g'(x)=\frac{x}{\sqrt{x^2-9}}$

$g'(5)=\frac{5}{\sqrt{5^2-9}}$

$\boxed{g'(5)=\frac{5}{4}}$

$f(x)=\sqrt{x^2+9}$ | $g(x)$

$(c,5)$ | $(5,c)$

$5=\sqrt{c^2+9}$

$25=c^2+9$

$16=c^2$

$4=c$

$f'(x)=\frac{x}{\sqrt{x^2+9}}$

$f'(4)=\frac{4}{\sqrt{4^2+9}}=\frac{4}{5}$

$g'(5)=\frac{1}{f'(4)}$

$\boxed{g'(5)=\frac{5}{4}}$