Key 1617

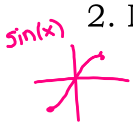


## Do Now:

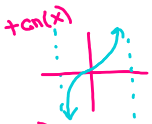
1. Evaluate the following expressions without using a calculator.

a. $\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$ b. $\arcsin(1) = \frac{\pi}{2}$ c. $\arctan(-1) = -\frac{\pi}{4}$

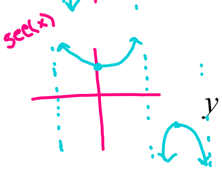
2. Now, write the domain and range of each inverse function below.




$y = \sin^{-1}(x)$ Domain: $[-1, 1]$ Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$




$y = \tan^{-1}(x)$ Domain: $(-\infty, \infty)$ Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



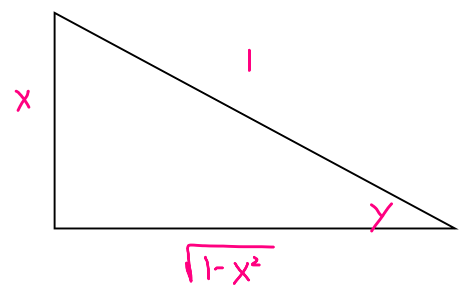

$y = \sec^{-1}(x)$ Domain: $(-\infty, -1] \cup [1, \infty)$ Range: $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

## Class Work:

Find $\frac{dy}{dx}$ for $y = \arcsin(x)$ $\Rightarrow$ $\sin y = x$

$\cos y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$

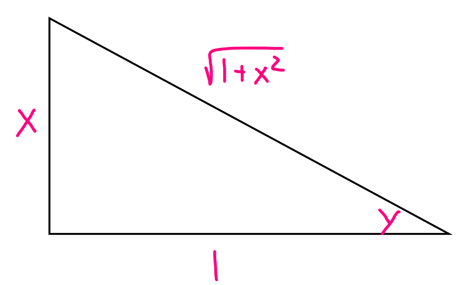



$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$

Find $\frac{dy}{dx}$ for $y = \arctan(x)$ $\Rightarrow$ $\tan y = x$

$\sec^2 y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+x^2}$




$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$

Find $\frac{dy}{dx}$ for $y = arc\sec(x)$ $\Rightarrow \sec y = x$

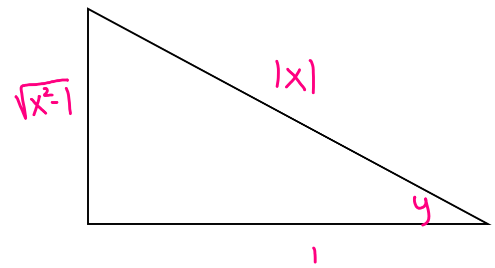
$$\cos y = \frac{1}{x}$$

$$-\sin y \frac{dy}{dx} = -\frac{1}{x^2}$$

$$\frac{dy}{dx} = \frac{-1}{x^2 \sin y}$$

$$= \frac{-1}{x^2 \frac{\sqrt{x^2-1}}{|x|}}$$

$$= \frac{-1}{|x|\sqrt{x^2-1}}$$




$$\frac{d}{dx}\left(\sec^{-1}(x)\right) = \frac{-1}{|x|\sqrt{x^2-1}}$$

## Inverse Trigonometric Derivatives

$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$

$\frac{d}{dx}(arc\sec(x)) = \frac{1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(arc\cot(x)) = \frac{-1}{1+x^2}$

$\frac{d}{dx}(arc\csc(x)) = \frac{-1}{|x|\sqrt{x^2-1}}$

**Class Work Problems:**
Differentiate the following functions:

1. $y = \arccos(x^2)$

$$y' = \frac{-1}{\sqrt{1-(x^2)^2}} \cdot 2x = \frac{-2x}{\sqrt{1-x^4}}$$

2. $h(x) = \csc^{-1}\left(\frac{x}{3}\right)$

$$h'(x) = \frac{-1}{\left|\frac{x}{3}\right|\sqrt{\frac{x^2}{9}-1}} \cdot \frac{1}{3} = \frac{-1}{|x|\frac{\sqrt{x^2-9}}{3}} = \frac{-3}{|x|\sqrt{x^2-9}}$$

3. $y = 12\cos^{-1}(-2x)$

$$y' = 12\left(\frac{-1}{\sqrt{1-4x^2}}\right) \cdot -2 = \frac{24}{\sqrt{1-4x^2}}$$

4. $y = \arcsin(x) + x\sqrt{1-x^2}$

$$y' = \frac{1}{\sqrt{1-x^2}} + \sqrt{1-x^2} + x\left(\frac{-2x}{2\sqrt{1-x^2}}\right)$$

$$y' = \frac{1}{\sqrt{1-x^2}} + \sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}}$$

$$y' = \frac{1+1-x^2-x^2}{\sqrt{1-x^2}} = \frac{2-2x^2}{\sqrt{1-x^2}} = 2\sqrt{1-x^2}$$

5. $y = x(\arctan(x))^2$

$y' = (\arctan(x))^2 + \dfrac{x\,2\arctan(x)}{1+x^2}$

$y' = (\arctan(x))^2 + \dfrac{2x\arctan(x)}{1+x^2}$

6. $y = \left(\sin^{-1}(x^3)\right)^4$

$y' = 4(\sin^{-1}(x^3))^3\left(\dfrac{1}{\sqrt{1-x^6}} \cdot 3x^2\right)$

$y' = \dfrac{12x^2(\sin^{-1}(x^3))^3}{\sqrt{1-x^6}}$

7. $y = arc\cot\left(\dfrac{1}{x}\right) - \arctan(x)$

$y' = \dfrac{-1}{1+\left(\frac{1}{x}\right)^2} \cdot \left(\dfrac{-1}{x^2}\right) - \dfrac{1}{1+x^2}$

$= \dfrac{1}{x^2+1} - \dfrac{1}{1+x^2}$

$= 0$

8. $y = x\arcsin(x) + \sqrt{1-x^2}$

$y' = \arcsin x + \dfrac{x}{\sqrt{1-x^2}} + \dfrac{-2x}{2\sqrt{1-x^2}}$

$y' = \arcsin(x)$

9. A particle moves along the $x$-axis so that its position at any time $t \geq 0$ is $x(t) = \tan^{-1}\sqrt{t}$. What is the velocity of the particle when $t = 16$?

$x'(t) = v(t) = \dfrac{1}{1+t}\left(\dfrac{1}{2\sqrt{t}}\right) = \dfrac{1}{2\sqrt{t}(t+1)}$

$v(16) = \dfrac{1}{2\sqrt{16}(16+1)} = \dfrac{1}{136}$

10. Find an equation for the line tangent to the graph of $y = \cot^{-1} x$ at $x = -1$.

$y' = \dfrac{-1}{1+x^2}$

$y'|_{x=-1} = \dfrac{-1}{2}$

$y|_{x=-1} = \cot^{-1}(-1) = -\dfrac{\pi}{4}$

$y + \dfrac{\pi}{4} = -\dfrac{1}{2}(x+1)$

11. Find the derivative with respect to $x$ of each of the following functions:

a. $y = \arccos\left(\frac{1}{x}\right)$

$y' = \frac{-1}{\sqrt{1-\frac{1}{x^2}}} \cdot \frac{-1}{x^2}$

$= \frac{1}{x^2 \frac{\sqrt{x^2-1}}{|x|}}$

$= \frac{1}{|x|\sqrt{x^2-1}}$

b. $f(x) = \csc^{-1}(5x)$

$f'(x) = \frac{-1}{|5x|\sqrt{25x^2-1}} \cdot 5$

$= \frac{-1}{|x|\sqrt{25x^2-1}}$

c. $y = \tan^{-1}\left(\sqrt{x^2-1}\right) + \sin^{-1}(2x)$

$y' = \frac{1}{1+x^2-1} \cdot \frac{2x}{2\sqrt{x^2-1}} + \frac{1}{\sqrt{1-4x^2}} \cdot 2$

$= \frac{1}{x^2\sqrt{x^2-1}} + \frac{2}{\sqrt{1-4x^2}}$

d. $h(x) = \sqrt{x^2-1} - arc\sec(x)$

$h'(x) = \frac{2x}{2\sqrt{x^2-1}} - \frac{1}{|x|\sqrt{x^2-1}}$

$= \frac{x|x|-1}{|x|\sqrt{x^2-1}}$

e. $y = arc\cot\sqrt{x}$

$y' = \frac{-1}{1+x} \cdot \frac{1}{2\sqrt{x}}$

$y' = \frac{-1}{2\sqrt{x}(x+1)}$

f. $y = x\sin^{-1}(x) + \sqrt{1-x^2}$

$y' = \sin^{-1}(x) + \frac{x}{\sqrt{1-x^2}} + \frac{-2x}{2\sqrt{1-x^2}}$

$y' = \sin^{-1}(x)$

12. Find an equation of the tangent line to the graph of the equation at the given point:

a. $\arctan(xy) = \arcsin(x+y)$ , $(0,0)$

$\frac{1}{1+x^2y^2}(y+xy') = \frac{1}{\sqrt{1-(x+y)^2}} \cdot (1+y')$

$y'|_{(0,0)} = -1$

$y = -x$

b. $\arctan(x+y) = y^2 + \frac{\pi}{4}$, $(1,0)$

$\frac{1}{1+(x+y)^2}(1+y') = 2yy'$

$y'|_{(1,0)} = -1$

$y - 0 = -1(x-1)$

$y = -x+1$