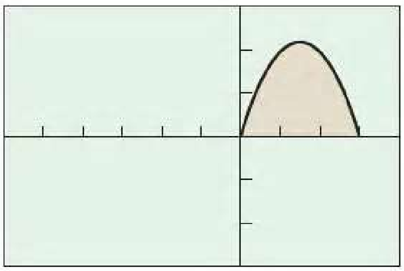
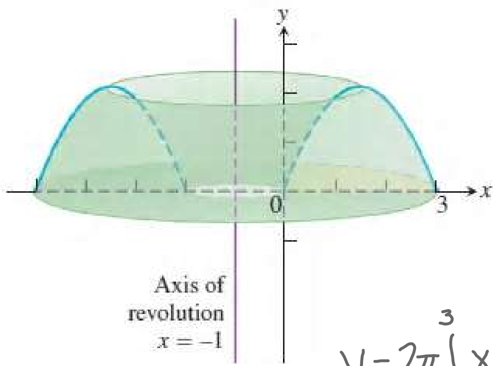
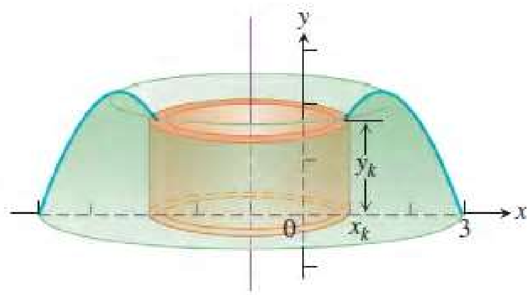
When the axis of revolution is **parallel** to the best choice for a representative rectangle, the **Shell Method** can be used to find the volume of a solid of revolution. Instead of summing the volume of thin slices (washers or disks), we sum volumes of thin cylindrical shells that grow outward from the axis of revolution like tree rings.

Example 1: The region enclosed by the *x*-axis and $f(x)=3x-x^2$ is revolved about the line $x=-1$ to generate the shape of a bundt cake. Find the volume of the cake.





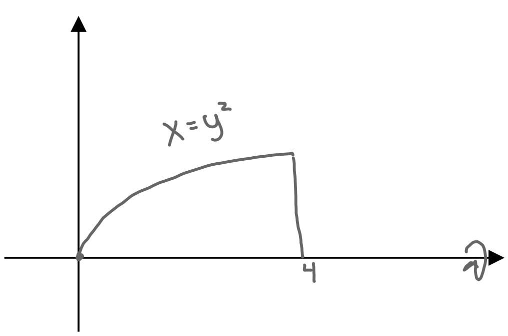


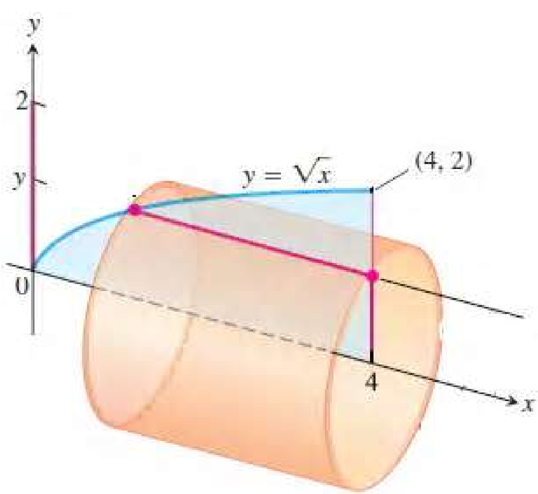
$$V=2\pi\int_0^3 x(3x-x^2)\,dx$$

Each shell has a unique height, a distance from the axis of symmetry (radius), and a small thickness.

Volume by Shells: $2\pi\int_a^b(\textit{Shell radius})(\textit{Shell Height})dx$ or $2\pi\int_c^d(\textit{Shell radius})(\textit{Shell Height})dy$

Example 2: The region bounded by $y=\sqrt{x}$, the *x*-axis, and $x=4$ is revolved about the *x*-axis to generate a solid. Find the volume using the shell method. Check with another method.





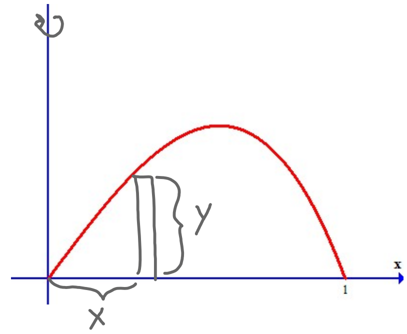
$$2\pi\int_0^2 y(4-y^2)\,dy=8\pi$$

$$\pi\int_0^4 (\sqrt{x})^2\,dx=8\pi$$

Class Work:

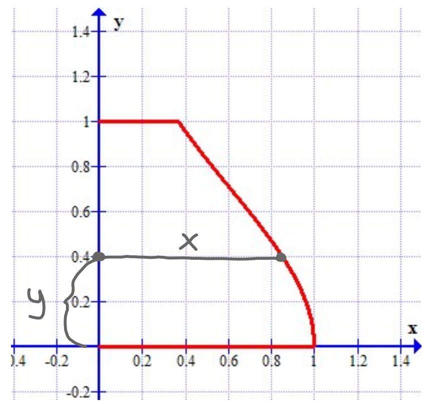
1. Find the volume of the solid of revolution formed by revolving the region bounded by $y = x - x^3$ and the x-axis $(0 \le x \le 1)$ about the y-axis.

$$V = 2\pi \int_0^1 x(x - x^3)\,dx = \frac{4\pi}{15}$$

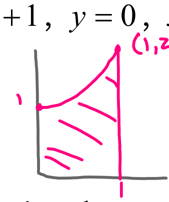


2. Find the volume of the solid of revolution formed by revolving the region bounded by the graph of $x = e^{-y^2}$ and the y-axis $(0 \le y \le 1)$ about the x-axis.

$$V = 2\pi \int_0^1 y e^{-y^2}\,dy = \pi\left(1 - \tfrac{1}{e}\right)$$

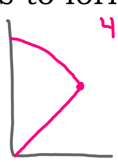


4. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis.



$$V = 2\pi \int_0^1 x(x^2 + 1)\,dx = \frac{3\pi}{2}$$

5. The region bounded by the curves $y = 4 - x^2$, $y = x$, and $x = 0$ is revolved about the y-axis to form a solid. Use cylindrical shells to find the volume of the solid.



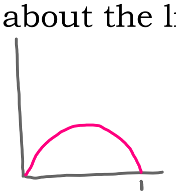
$4 - x^2 = x$

$0 = x^2 - x - 4$

$x \approx 1.562$

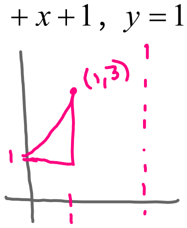
$$V = 2\pi \int_0^{1.562} x(4 - x^2 - x)\,dx = 13.327$$

6. Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.



$$V = 2\pi \int_0^1 (2 - x)(x - x^2)\,dx = \frac{\pi}{2}$$

7. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3 + x + 1$, $y = 1$, and $x = 1$ about the line $x = 2$.



$$V = 2\pi \int_0^1 (2 - x)(x^3 + x + 1 - 1)\,dx = \frac{29\pi}{15}$$