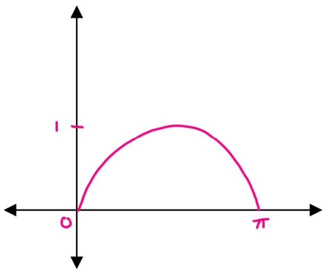
7.3 day 2 & 3

Tuesday, May 29, 2018 1:51 PM

AP Calculus AB **Section 7.3:** The Shell Method **Day 2 & 3**

Do Now: Calculator Active!
Consider the region R bounded by the function $y = \sin(x)$, $x = 0$, $x = \pi$ and $y = 0$. Sketch the region and indicate the point(s) of intersection. Use the disk, washer or shell method to set-up and solve an integral that finds the volume generated when R is revolved about:



a) the line $y = 0$

$V = \pi \int_0^{\pi} (\sin x)^2 dx \approx 4.935$

b) the line $x = 0$

$V = 2\pi \int_0^{\pi} x(\sin x) dx \approx 19.739$

c) the line $y = -1$

$V = \pi \int_0^{\pi} (\sin x + 1)^2 - (1)^2 dx \approx 17.501$

d) the line $x = \pi$

$V = 2\pi \int_0^{\pi} (\pi - x)\sin x \, dx \approx 19.739$

e) the line $y = 4$

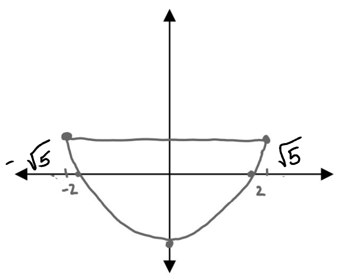
$V = \pi \int_0^{\pi} 4^2 - (4 - \sin x)^2 dx \approx 45.331$

f) the line $x = -\pi$

$V = 2\pi \int_0^{\pi} (\pi + x)\sin x \, dx \approx 59.218$

Class Work: Calculator Active!

1. Consider the region R bounded by the functions $y = x^2 - 4$ and $y = 1$ ~~$y = 1 - x^2$~~. Sketch the region and indicate point(s) of intersection. Find the volume generated when R is revolved about:



a) the line $x = 3$

$2\pi \int_{-\sqrt{5}}^{\sqrt{5}} (1 - (x^2 - 4))(3 - x) dx = 280.993$

b) the line $x = -4$

$2\pi \int_{-\sqrt{5}}^{\sqrt{5}} (1 - (x^2 - 4))(x + 4) dx = 374.657$

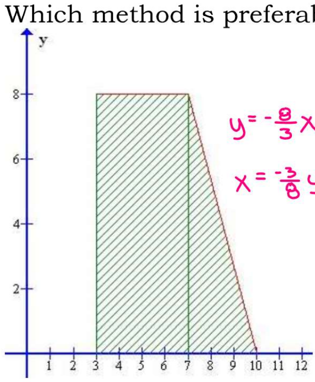
c) the line $y = 2$

$\pi \int_{-\sqrt{5}}^{\sqrt{5}} (2 - (x^2 - 4))^2 - (2 - 1)^2 dx = 280.993$

d) the line $y = +1$

$\pi \int_{-\sqrt{5}}^{\sqrt{5}} (1 - (x^2 - 4))^2 dx = 187.328$

2. Consider the region R as shown below. This region is revolved around the y-axis to create a solid. Find the volume using **both** the disk/washer and the shell method. Which method is preferable? Why?



$y = -\frac{8}{3}x + \frac{80}{3}$

$x = -\frac{3}{8}y + 10$

A. Disk/washer

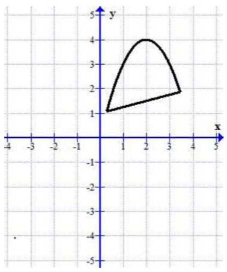
$V = \pi \int_0^8 (-\frac{3}{8}y + 10)^2 - (3)^2 dy = 512\pi$

B. Shell

$V = 2\pi \int_3^7 x(8) dx + 2\pi \int_7^{10} x(-\frac{8}{3}x + \frac{80}{3}) dx = 512\pi$

3. Consider the region bounded by the functions $y=\frac{x}{4}+1$ and $y=4x-x^2$. Set-up and solve an integral using cylindrical shells that represents the volume generated when the above region is revolved about:

a) the y-axis b) the line $x=5$ c) the x-axis d) the line $x=-2$



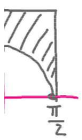
a. $V=2\pi\int_{.289}^{3.461} x\left(4x-x^2-\frac{x}{4}-1\right)dx \approx 62.674$

b. $V=2\pi\int_{.289}^{3.461}(5-x)\left(4x-x^2-\frac{x}{4}-1\right)dx \approx 104.457$

c. $V=\pi\int_{.289}^{3.461}(4x-x^2)^2-\left(\frac{x}{4}+1\right)^2dx \approx 82.730$

d. $V=2\pi\int_{.289}^{3.461}(x+2)\left(4x-x^2-\frac{x}{4}-1\right)dx \approx 129.527$

4. Consider the region defined by $y=\cos(x)$, $y=2$, $x=0$ and $x=\frac{\pi}{2}$. Set-up and solve an integral using cylindrical shells that represents the volume generated when the above region is revolved about:



a) the y-axis b) the line $x=\frac{\pi}{2}$ c) the line $y=2$ d) the line $y=-2$

a) $2\pi\int_0^{\frac{\pi}{2}} x(2-\cos x)dx \approx$ 11.917

b) $2\pi\int_0^{\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)(2-\cos x)dx \approx$ 9.220

c) $2\pi\int_0^1(2-y)\left(\frac{\pi}{2}-\arccos y\right)dy + 2\pi\int_1^2(2-y)\left(\frac{\pi}{2}\right)dy \approx$ 9.640

d) $2\pi\int_0^1(y+2)\left(\frac{\pi}{2}-\arccos y\right)dy + 2\pi\int_1^2(2+y)\left(\frac{\pi}{2}\right)dy$ 44.184

e) The line $x=c$ $\left(0<c<\frac{\pi}{2}\right)$ is added to the region. When this new region is revolved about the y-axis, the resulting volume is exactly half of the volume generated in part a. Find the value of c. Round to three decimal places.

$2\pi\int_0^c x(2-\cos x)dx \approx 5.9585$

$c \approx 1.194$

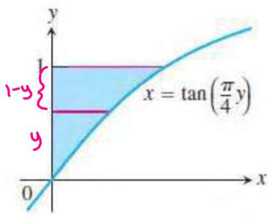
*enter in calc on $(0,\frac{\pi}{2})$ window

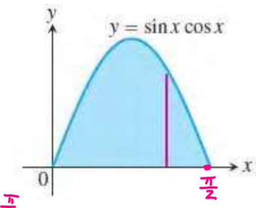
* very slow to graph!

5. Set up, but do not solve, an integral that finds the volume of the solid generated by revolving the function around the given axes.

a) y-axis, x-axis and $y = 1$

b) x-axis, the y-axis, $x = 2\pi$ & $x = -1$





sinxcosx = 0
sinx = 0   cosx = 0
x = 0, π $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\pi\int_0^1 (\tan(\frac{\pi}{4}y))^2 dy$

c. $2\pi\int_0^1 (1-y)\tan(\frac{\pi}{4}y)dy$

$\pi\int_0^1 y(\tan(\frac{\pi}{4}y))dy$

a. $\pi\int_0^{\frac{\pi}{2}} (\sin x\cos x)^2 dx$

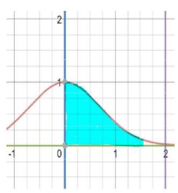
b. $2\pi\int_0^{\frac{\pi}{2}} x(\sin x\cos x)dx$

c. $2\pi\int_0^{\frac{\pi}{2}} (2\pi - x)(\sin x\cos x)dx$

d. $2\pi\int_0^{\frac{\pi}{2}} (x+1)(\sin x\cos x)d$

6. Find the volume of the following solids formed by revolving the region bounded by $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = 2$ about the appropriate axis of revolution. Use the integration capabilities of the graphing calculator to find the volume.

a) x-axis b) $y = -2$ c) $x = -2$ d) $x = 3$ e) y-axis



a. $\pi\int_0^2 (e^{-x^2})^2 dx = 1.969$

b. $\pi\int_0^2 (e^{-x^2}+2)^2 - (2)^2 dx = 13.053$

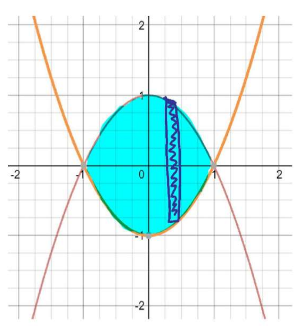
c. $2\pi\int_0^2 (e^{-x^2})(x+2)\,dx = 14.169$

d. $2\pi\int_0^2 (3-x)e^{-x^2}dx = 13.563$

e. $2\pi\int_0^2 xe^{-x^2}dx = 3.084$

7. Consider the region R bounded by the functions $y = x^2 - 1$ and $y = 1 - x^2$. Sketch the region & indicate point(s) of intersection. Set-up, but do not solve, an integral that finds the volume generated when R is revolved about:

a) $x = 3$ b) $x = -4$ c) $y = 2$ d) $y = -1$ e) $x = 1$



$x^2 - 1 = 1 - x^2$
$2x^2 = 2$
$x^2 = 1$
$x = \pm 1$

a. $2\pi\int_{-1}^{1} (3-x)(1-x^2-x^2+1)dx = 2\pi\int_{-1}^{1} (3-x)(2-2x^2)dx$

b. $2\pi\int_{-1}^{1} (x+4)(1-x^2-x^2+1)dx = 2\pi\int_{-1}^{1} (x+4)(2-2x^2)dx$

c. $\pi\int_{-1}^{1} (2-x^2+1)^2 - (2-1+x^2)^2 dx = \pi\int_{-1}^{1} (3-x^2)^2 - (1+x^2)^2 dx$

d. $\pi\int_{-1}^{1} (1-x^2+1)^2 - (x^2-1+1)^2 dx = \pi\int_{-1}^{1} (2-x^2)^2 - (x^2)^2 dx$

e $2\pi\int_{-1}^{1} (1-x)(1-x^2-x^2+1)dx = 2\pi\int_{-1}^{1} (1-x)(2-2x^2)dx$